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Brane Worlds, the Cosmological Constant and String Theory

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Abstract: We argue that traditional methods of compactification of string theory make it very difficult to understand *how* the cosmological constant becomes zero. String inspired models can give zero cosmological constant after fine tuning but since string theory has no free parameters it is not clear that this is allowed. Brane world scenarios on the other hand while they do not answer the question as to *why* the cosmological constant is zero do actually allow a choice of integration constants that permit flat four space solutions. In this paper we discuss gauged supergravity realizations of such a world. To the extent that this starting point can be considered a low energy effective action of string theory (and there is some recent evidence supporting this) our model may be considered a string theory realization of this scenario.

Keywords: D-branes, Superstring Vacua, Supergravity Models..

Contents

1. Introduction

The traditional method of compactification of superstring theory involves writing the ten dimensional manifold as a direct product of a non-compact Minkowskian four manifold and a compact Euclidean six manifold $M_{10} = M_4 \otimes M_6$, with the size of the compact space being set by the string scale. The four manifold is identified with the space we observe around us and is therefore taken to be flat. The compact manifold is taken to be a Ricci flat Kähler manifold (i.e. a Calabi-Yau space or an orbifold) so that we obtain $\mathcal{N} = 1$ supersymmetry (SUSY) in the four manifold [1]. The latter is required for at least two reasons. The first is the so-called hierarchy problem involving the stabilization of the weak scale to quantum corrections. With (softly broken) $\mathcal{N} = 1$ SUSY one can protect this hierarchy. The other issue is that of the cosmological constant (CC). Again supersymmetry offers the hope of stabilizing this against quantum corrections i.e. if the ultraviolet theory (say at the string scale) has zero cosmological constant then as long as SUSY is preserved it will remain zero. In this case case, however, once SUSY is broken, it is difficult to see how one can prevent a CC, at least of order the mass splitting between fermions and bosons, from being generated.

Actually getting a realistic model where this is achieved would be progress, since there would also be standard model phase transitions which are also of the same order and could in principle cancel the SUSY breaking effects. Nevertheless at this point it is, we believe, fair to say that even this has not yet been achieved in a natural way, since typical cosmological constants that are generated upon SUSY breaking are at an intermediate scale of $\mathcal{O}(10^{11} GeV)$ (which is the actual scale of SUSY breaking in the so-called hidden sector).

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To get a zero (or small) CC requires fine tuning of some parameter in the scalar potential of $\mathcal{N}=1$ supergravity. However string theory has no tunable parameters. In fact it is not very clear how to even generate the potential for the moduli (the dilaton, the size and shape of the compact manifold) but it is generally agreed that, either stringy or low energy field theoretic, non-perturbative effects will give such a potential and stabilize these moduli. [1]

Let us first think about the field theoretic stabilization mechanism. One possibility is to have several gauge groups in the hidden sector whose gauginos condense. This is the so-called race track model. The effective superpotential needs to be a sum of three exponentials of the moduli fields, two are needed to get a weak gauge coupling minimum and an intermediate scale of SUSY breaking and a third is needed in order to fine tune the cosmological constant to zero. This model has various problems associated with the steepness of the potential (see [2] for a recent discussion) but even apart from that it seems unlikely that the sort of fine tuning that is required is allowed by string theory. An alternative would be to have the moduli stabilized by string scale physics. In this case however one needs to find field theoretic models which solve the practical cosmological constant problem i.e. obtain a model with CC only at the weak scale as advocated in [2]. Even this seems a non-trivial task; in particular, it seems that one needs a constant in the superpotential to do so and to get a zero (or small) cosmological constant would seem to require a fine tuning that would really be at variance with what one expects from string theory. Similar remarks apply to all models proposed so far for getting standard model physics out of string theory (including many brane world type scenarios based on type I string theory) once one tries to get flat four dimensional space after SUSY breaking.

The main point of this paper is to propose a string theoretic brane world scenario for getting a zero cosmological constant based on arguments sketched in [3]. This would imply a radical departure from what happens in the standard string compactification models discussed in the above paragraphs. Instead of compactifying to four flat dimensions with $\mathcal{N}=1$ supersymmetry, we compactify the ten dimensional theory (here we will concentrate on type IIB) on a five sphere (or squashed sphere) to five dimensions [4]. In contrast to the standard compactifications we will now have a potential for at least the T-moduli (i.e. the size and shape of the squashed sphere) and of course a bulk cosmological constant. What we have is five dimensional gauged supergravity but the four dimensional world, instead of being a further compactification of this, is viewed as a brane sitting in this five dimensional bulk. As discussed in [3], there are various possibilities for getting flat four dimensional space on the brane without fine tuning[2] none of which explain why the cosmological

[1]For a recent discussion highlighting the problems involved with references to earlier literature see [2].

[2]For a discussion of a case which apparently does not involve a choice of integration constant but involves a bulk singularity see the first paper of [5]. Apart from the difficulty of interpreting

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constant is zero. But unlike in the standard compactifications, where it is difficult to understand even how is the CC tuned to zero, in our case we believe there is a viable scenario whereby with a choice of integration constants and a compactification parameter (which is not determined by the ten dimensional theory) flat space four dimensional solutions can be obtained.

Although our discussion is confined to an analysis of a five dimensional effective action coming from compactifying a ten dimensional action for type IIB supergravity on a (squashed) sphere, we believe that is quite likely that there is a string construction in such a Ramond-Ramond (RR) background. In fact, recently there has been considerable progress in constructing such theories [7]. Thus we think that the model that we propose here could be promoted to the complete string theory.

Another issue that we should discuss here is that of singular brane configurations versus smooth configurations. We take the point of view that our branes are D-branes and/or orientifold planes so that as far as the low energy effective action is concerned their thickness is infinitesimal.[3] Their transverse size in other words is set by the string scale and to the extent that we are working in the low energy effective action we can ignore their thickness. Thus we will be looking for solutions to the bulk equations in the presence of these branes, which will act as singular source terms as for instance in section 3 of [8]. We believe that the considerations of [9] where the authors look for brane scenarios that try to mimic the Randall - Sundrum (RS) scenarios [10] and/or [11] without putting in sources (and finding a negative result) are not relevant to our construction. On either side of our branes we will have smooth solutions to the supergravity equations which will then be matched at the position of the branes. It should also be pointed out here that a model for the real world would in fact require a brane which carried gauged degrees of freedom and so we really would want to think in terms of D-branes rather than smooth brane like configurations. The picture that we have (i.e. basically that in [10],[11]) actually implies the physical existence of a brane (on which the standard model should live) placed in the bulk supergravity space.

Our model has the explicit D-brane/orientifold planes at the fixed points of the $\frac{S^1}{Z_2}$ orbifold in the (x^4) fifth direction. We assume that the ten dimensional dilaton is frozen by string scale dynamics. When the squashing modes are constant but at a certain critical point with non-zero values, the bulk is a $\mathcal{N}=2$ background. Turning on the breathing mode φ of the sphere should not affect the supersymmetry. The supersymmetry of the D/orientifold 3-plane would be $\mathcal{N}=1$. This should give a

this naked singularity (which is common to all the models in both papers of [5] this model has the problem that the brane tension chosen is not renormalization group (RG) invariant. For comments on these models along these lines see [3], [6].

[3]We stress that although flat space considerations imply that gravity cannot be confined to a D-brane (as one wants for a one brane RS type scenario) in a warped background this is not necessarily the case.

solution of the RS type without fine tuning and with $\mathcal{N}=1$ supersymmetry. After supersymmetry breaking however one does not expect a solution for arbitrary values of the radius of the S^1. In this case, in addition to the radius of the circle, there are also two integration constants from the metric factor and breathing mode and the constant coming from the compactification, - four constants in all that will be determined by the two pairs of matching conditions at each brane. Thus all the constants are completely fixed.

As stressed in [3], this is of course not a solution to *the* cosmological constant problem since there is no explanation of why the integration constants are chosen to give the values that they need to have in order to get flat branes. Nevertheless we believe it is progress in that at least there is an explanation as to how one might get a zero CC in a string theoretic scenario after SUSY breaking.[4]

It should also be pointed out that the brane world scenario appears to be the only context, within string theory, that the idea of using the free parameter (from the point of view of the ten dimensional theory) in the potential of gauged supergravity, to adjust the cosmological constant to zero, can be made use of. This is because this mechanism can only be used in type IIA or IIB theories (and orientifolds thereof) and the standard model/real world in such theories necessarily lives on a D-brane.

In the next section we discuss a possible string theory set up in which the scenario we have in mind may be realized. In section 3 we discuss special bulk solutions to the five dimensional equations and in section 3.1 we display approximate general solutions with all the allowed integration constants that are necessary to obtain a solution with two branes. In section 4 we summarize the physics of the construction and its significance, and then speculate on possible phenomenological applications.

2. The String Theory Setup

We wish to consider type IIB string theory compactified on a five sphere or squashed sphere. Typically this background is obtained by turning on the five form RR flux. We assume that string theory on such a background exists perhaps along the lines discussed by Berkovits et al [7]. If this is indeed justified, it should follow that the entire D-brane machinery can be taken over.

Thus we look at S^1/Z_2 orientifold of the five dimensional theory resulting from the (squashed) sphere compactification with S^1 being a circle of radius R and the Z_2 action being $x^4 \to -x^4$. The theory is a five dimensional gauged supergravity which contains in addition to the dilaton-axion, 40 other scalars some of which can be interpreted as squashing modes of the five sphere. These fields are massless

[4]While this paper was being prepared for publication a paper which discusses an alternate scenario for two flat branes without fine tuning appeared. Our mechanism is replaced there with an assumption about supersymmetry in the bulk and on the "Planck brane" [12]. However we do not understand how a fine tuning in the bulk potential is avoided in this case.

in the limit when the gauge coupling is zero. In addition there is the so called (compactification scale) "breathing mode" and of course the gauge fields which along with the two 2-form fields associated with the F and D strings will be set to zero in the following. Also we assume that the ten dimensional dilaton is frozen by string scale dynamics.

Let us first consider the round sphere compactification. The ten dimensional metric is written as

$$ds_{10}^2 = e^{2\alpha\varphi} ds_5^2 + e^{2\beta\varphi} ds^2(S^5)$$

with $\alpha = \frac{1}{4}\sqrt{\frac{5}{3}}$, $\beta = -\frac{3}{5}\alpha$. The ansätz for the self-dual 5 - form is

$$H_{(5)} = 4me^{8\alpha\varphi}\epsilon_{(5)} + 4m\epsilon_{(5)}(S^5), \tag{2.1}$$

where $\epsilon_{(5)}$ and $\epsilon_{(5)}(S^5)$ are the volume forms of the non-compact and compact spaces respectively. The effective five dimensional action is then [4]

$$S = \int d^5x\sqrt{-G_5}[\mathcal{R} - \frac{1}{2}(\partial\varphi)^2 + e^{\frac{16\alpha\varphi}{5}} R_5 - 8m^2 e^{8\alpha\varphi}], \tag{2.2}$$

where R_5 is the scalar curvature of the five sphere and we have set to zero all fields that are irrelevant to our discussion. Note [4] that this action allows the AdS_5 solution with constant breathing mode $\varphi = \varphi_0$ given by

$$e^{\frac{24\alpha}{5}\varphi_0} = \frac{R_5}{20m^2} \tag{2.3}$$

and

$$R_{MN} = -4m^2 e^{8\alpha\varphi_0} G_{5MN}. \tag{2.4}$$

Now the fifth dimension is a $\frac{S^1}{Z_2}$ orbifold. The fixed points would be orientifold planes and we place D-branes at one or other (or both) fixed points so as to have two (composite) branes at the ends of the fifth dimension. The picture is just like that in the ten dimensional type IA situation analyzed by Polchinski and Witten [13]. In the original ten dimensional framework, the self dual five form field strength would satisfy in the presence of a collection of n_0 branes located near $x^4 = 0$ and extending in the x^1, x^2, x^3 directions (counting -16 for the orientifold fixed plane), the equation $dH_{(5)} = n_0\tau\delta(x^4) \wedge dx^4 \wedge \epsilon_5(S_5)$, where τ is the charge of a single brane. We take x^0 to be the time like direction and $x^5, \cdots, x^9$ the directions along the five sphere. Substituting expression (2.1) into this, gives $\partial_{x^4} m(x^4) = n_0\tau\delta(x^4)$. The solution for $H_{(5)}$ now takes the form as before, i.e. (2.1), but with m being now a piece wise continuous function of x^4 with a jump $\Delta m = n_0\tau$ at $x^4 = 0$ and a similar jump at $x^4 = \pi R$. Using also the symmetry under $x^4 \to -x^4$, we find the sort of situations illustrated in figs. 1, 2, 3.

Note that the quantization of H_5 also follows directly from its coupling to the $D3$ brane. Consistent coupling requires by a standard argument $\tau \int_{S_5} H_5 = 2\pi n$ where n

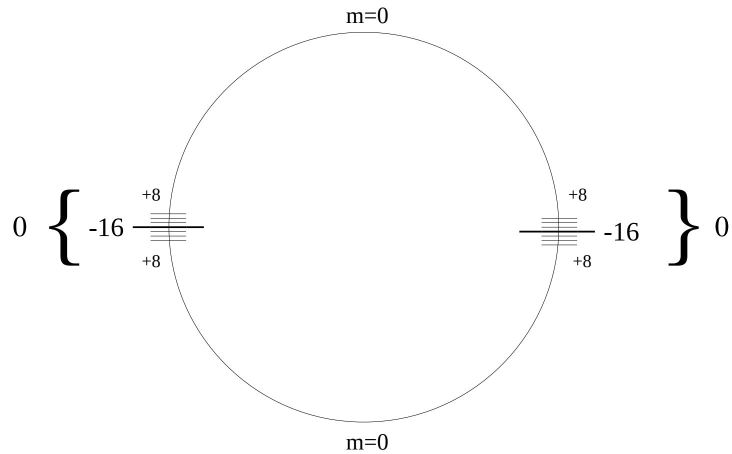

Figure 1: IIB on S^5 brane world 1: $SO(16) \times SO(16)$.

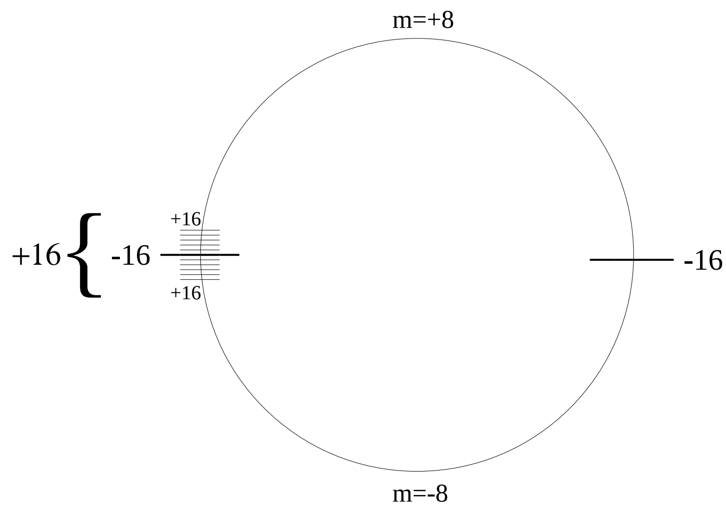

Figure 2: IIB on S^5 brane world 2: $SO(32)$.

is an integer and $\tau = 2\pi M_s^4$ is the D_3 brane charge in terms of the string scale M_s. Using equation (1) for H_5 and $\int_{S_5} = \pi^3 r^5$ we then get

$$m = n \left(\frac{1}{M_s^4 4\pi^3 r^5} \right).$$

The units for m given in the figures is then the quantity in parantheses in the above equation.

In the above, we should point out that we have assumed not only that there is a valid string description as in Berkovits et al [7] but also that the corresponding five dimensional orientifolds have charges -16. These values are just given for illustrative purposes and of course if the general scenario is valid with however different charges, then the examples would have to be modified accordingly. We should also point out that alternatively, one might try to justify such a picture by considering the following

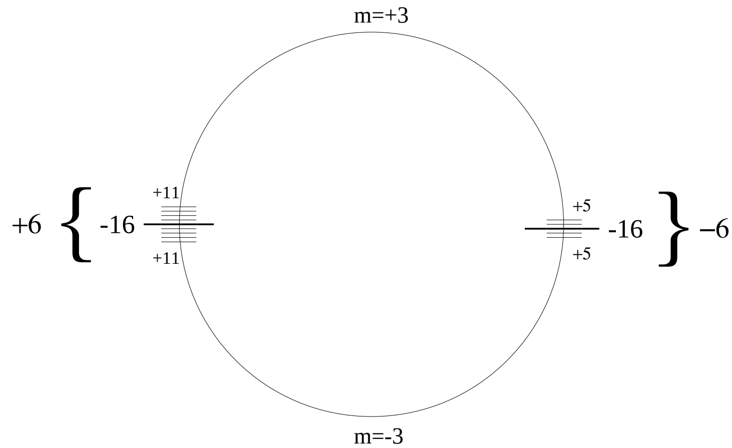

Figure 3: IIB on S^5 brane world 3: $SO(22) \times SO(10)$.

procedure. First, compactify type I string theory on a five torus. Taking the T-dual picture one has a type IIB orientifold with 2^5 orientifold 3-planes at the fixed points of the Z_2 orbifold symmetry of the dual torus and 32 D3-branes to cancel the tadpoles. Now let the size of the dual torus go to infinity while keeping some number of the D-branes at the fixed point at the origin. Now adding the point at infinity should give presumably an S^5/Z_2. It is not quite clear to us which compactification makes sense in the complete string theory but given that the 5D gauged supergravity models appear to come from five sphere or squashed sphere compactifications we will just consider these, assuming that a theory along the lines of Berkovits et al [7] will allow our scenario.

As we will see below, the bulk is taken to be at the $\mathcal{N} = 2$ critical point of the gauged supergravity potential. In the presence of the branes this is then broken down to $\mathcal{N} = 1$ so that one may have a phenomenologically acceptable model on one or the other brane. Let us pick the one at $x^4 = 0$ to situate the standard model. The other one can then play the role of a hidden sector (as in the Horava-Witten theory [14]) where supersymmetry can be broken and then communicated to the visible brane. The only new point here is that any cosmological constant that is generated as a result of the SUSY breaking is compensated by adjustment of integration constants and R_5.

It should be remarked here that R_5 is on the same footing as the other integration constants (which occur in the five dimensional theory) from the point of view of the ten dimensional theory. It is not a modulus. The relevant modulus is the field φ. R_5 is, from the point of view of the ten dimensional theory, and presumably of string theory, an integration constant appearing in the ansatz for the ten dimensional metric. For compactification on a Ricci flat metric such as a Calabi-Yau metric, this constant can be absorbed into φ. In our case as long as m is non-zero i.e. the potential is not runaway, R_5 cannot be absorbed into the modulus φ. We stress again that

this is not a mechanism that was available with standard compactification scenarios (Calabi-Yau, Orbifold etc).[5]

The next issue is what is the appropriate 5D effective theory. Our choice is the $\mathcal{N}=2$ supersymmetric vacuum of [16] which corresponds to two non trivial scalars relaxed to their critical values plus a non trivial breathing mode, a generalization of the model with one squashing mode and a non trivial breathing mode model of [4]. The scalar potential of this model is expected to have a contribution from the curvature of the compact space and a number of contributions from non-trivial 5-form and 3-form fluxes. For simplicity, we will truncate the potential only to the curvature term and the 5-form flux term since these are sufficient to demonstrate our point. A more complete treatment should include all the other terms. The potential is then just as in the round sphere case (2.2), and it is:

$$V(\varphi) = -R_5 e^{a\varphi} + 8m^2 e^{b\varphi}. \tag{2.5}$$

3. Flat Domain Wall Solutions - Brane Worlds

The equations of motion corresponding to the action (2.2) with $\varphi = \varphi(x^4)$ and the flat four dimensional domain wall $5D$ metric

$$ds_5^2 = e^{2A(x^4)}\eta_{\mu\nu}dx^\mu dx^\nu + \left(dx^4\right)^2, \tag{3.1}$$

where $\eta_{\mu\nu} = diag(-1,+1,+1,+1)$, are

$$\varphi'' + 4A'\varphi' = \frac{\partial V(\varphi)}{\partial\varphi}, \tag{3.2}$$

$$A'' = -\frac{1}{6}\varphi'^2, \tag{3.3}$$

$$A'^2 = -\frac{1}{12}V(\varphi) + \frac{1}{24}\varphi'^2. \tag{3.4}$$

The prime denotes differentiation with respect to x^4. We emphasize that our domain wall ansatz is that we are looking for solutions with flat four dimensional slices of the five dimensional geometry.

It has been shown that by introducing a "superpotential" W, the second order differential equations reduce to [8], [17]:

$$\varphi' = \frac{\partial W(\varphi)}{\partial\varphi}, \tag{3.5}$$

[5] Recently, other warped compactifications, in the context of M/F theory, have appeared [15] where a RS type scenario with explicit branes is constructed. However for these Calabi-Yau compactifications with G-flux, it is not possible to use our mechanism, since the compact space is Ricci flat, so one does not have the required freedom in the choice of constants to cancel the CC.

$$A' = -\frac{1}{6}W, \tag{3.6}$$

$$V(\varphi) = \frac{1}{2}\Big(\frac{\partial W}{\partial \varphi}\Big)^2 - \frac{1}{3}W^2. \tag{3.7}$$

This is a system of decoupled first order differential equations instead of the coupled second order differential equations that we had before. A particular ansatz for W that satisfies the last condition for $V(\varphi)$ of the form (2.5), is

$$W = p_1 e^{\frac{a}{2}\varphi} + p_2 e^{\frac{b}{2}\varphi}, \tag{3.8}$$

with

$$p_1 = \pm 2\sqrt{\frac{2R_5}{a(b-a)}} \quad \text{and} \quad p_2 = \pm 2\sqrt{\frac{16m^2}{b(b-a)}}. \tag{3.9}$$

This ansatz essentially solves the system, but it seems that we have to pay a price. Namely, we have lost one integration constant because (3.8) is a particular solution of (3.7) and does not have the integration constant that a general solution should have.

Note that the original second order system apparently has four integration constants. However, (3.4) is a constraint equation which gives one relation between these constants. Also the zero mode of A can always be absorbed in a rescaling of coordinates. Thus the original system has two independent integration constants. As stressed by [8], the first order system must be completely equivalent to the second order system and the parameter space should have the same dimension. This is of course possible only if we use the general solution for W in which case the first order system will also have two independent integration constants, one in the solution for W and one in the solution for φ. Of course if we work with the particular solution (3.8), we would be restricting ourselves to a one parameter subspace of the solution space. We will come back to the problem of the lost integration constant later.

For the moment, we have to solve the following system of first order differential equations:

$$\varphi' = \frac{1}{2}ap_1 e^{\frac{a}{2}\varphi} + \frac{1}{2}bp_2 e^{\frac{b}{2}\varphi}, \tag{3.10}$$

$$A' = -\frac{1}{6}p_1 e^{\frac{a}{2}\varphi} - \frac{1}{6}p_2 e^{\frac{b}{2}\varphi}. \tag{3.11}$$

Integrating (3.10) gives the Lerch transcendent [18] as the solution,

$$r + c_1^{\varphi} = \frac{2}{ap_1}\sum_{n=0}^{\infty}\frac{(-\frac{bp_2}{ap_1})^n}{(a-b)n+a}\Big[1 - e^{\big(n(b-a)-a\big)\varphi}\Big], \tag{3.12}$$

which is quite hard to invert for general φ, so as to obtain an exact solution for the warp factor. It is much easier to work in certain interesting limits:

- $\varphi \to \varphi_0$.

  Let us first solve (3.10) and (3.11) for $\varphi = \varphi_0$, the value of φ at the critical point of V. We have $\varphi' = 0$ and therefore $\frac{\partial W}{\partial \varphi} = 0$, which implies through (3.7) that this vacuum characterizes a critical point (in fact a minimum) of the scalar potential. Furthermore, from (3.11) we see that it corresponds to an exact AdS vacuum, since A has a linear dependence in x^4. The condition, therefore, that determines φ_0, is

$$ap_1 e^{\frac{a}{2}\varphi_0} + bp_2 e^{\frac{b}{2}\varphi_0} = 0. \tag{3.13}$$

  Let us now solve the equations for a vacuum that is near the $\varphi = \varphi_0$ vacuum. For that, we assume

$$\varphi = \varphi_0 + \epsilon f(x^4), \tag{3.14}$$

  with ϵ a small number. Substituting this ansatz into (3.10), we find that f satisfies the differential equation $f' = 4kf$ and therefore

$$\varphi = \varphi_0 + c_1^{\varphi} e^{4kx^4}, \qquad k^2 = \frac{1}{32} a(b-a) R_5 e^{a\varphi_0}, \tag{3.15}$$

  where k and p_1 are of opposite signs and c_1^{φ} is the integration constant with ϵ absorbed in it. Solving (3.11) for A, yields the (almost AdS) warp factor:

$$A(x^4) = kx^4 + \mathcal{O}(\epsilon^2). \tag{3.16}$$

  The first term in the above, corresponds to the AdS part. The solution (3.15) and (3.16), however, is valid only for large $|x^4|$. In particular, if $k < 0$ (that is if $p_1 > 0$) then the solution is valid when $x^4 \to +\infty$ and if $k > 0$ ($p_1 < 0$) then the solution is valid when $x^4 \to -\infty$. Finally, (3.13) implies that p_1 and p_2 must come with opposite signs.

- $\varphi \to +\infty$.

  In this limit, we have

$$\varphi' \simeq \frac{1}{2} bp_2 e^{\frac{1}{2}b\varphi}, \tag{3.17}$$

  which can be integrated to give

$$\varphi = -\frac{2}{b}\Big[\ln\big(-\frac{b^2}{4}p_2 x^4 + c_1^{\varphi}\big)\Big], \tag{3.18}$$

  where c_1^{φ} is an integration constant. The above solution is valid for $-\frac{b^2}{4}p_2 x^4 + c_1^{\varphi} \to 0^+$. The solution for the warp factor turns out to be

$$e^{2A(x^4)} = \Big(-\frac{b^2}{4}p_2 x^4 + c_1^{\varphi}\Big)^{\frac{4}{3b^2}}, \tag{3.19}$$

  up to an (irrelevant) overall integration constant. Thus in this limit, we have $e^{2A} \to 0^+$.

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- $\varphi \to -\infty$.

  In this limit, we have

$$\varphi' \simeq \frac{1}{2} a p_1 e^{\frac{1}{2}a\varphi}, \tag{3.20}$$

  giving

$$\varphi = -\frac{2}{a}\Big[\ln\big(-\frac{a^2}{4}p_1 x^4 + c_1^\varphi\big)\Big], \tag{3.21}$$

  which implies that $-\frac{a^2}{4}p_1 x^4 + c_1^\varphi \to +\infty$ for any finite c_1^φ.

  The warp factor is

$$e^{2A(x^4)} = \Big(-\frac{a^2}{4}p_1 x^4 + c_1^\varphi\Big)^{\frac{4}{3a^2}}, \tag{3.22}$$

  up to the usual overall constant. Thus, we have $e^{2A} \to +\infty$ in this limit.

As was already noticed in [18], the above solution has two separate branches, one which corresponds to $\varphi \in (\varphi_0, +\infty)$ for $x^4 \in (-\infty, \frac{4}{b^2 p_2} c_1^\varphi)$ and one which corresponds to $\varphi \in (\varphi_0, -\infty)$ for $x^4 \in (-\infty, +\infty)$. We will see later that the separation of the solution in two distinct branches is probably special to our specific choice (3.8) of W and that this choice is equivalent to choosing a gauge where $c_2^\varphi = 0$. The first branch is singular at $x^4 = \frac{4}{b^2 p_2} c_1^\varphi$ because the warp factor vanishes, so we neglect it.

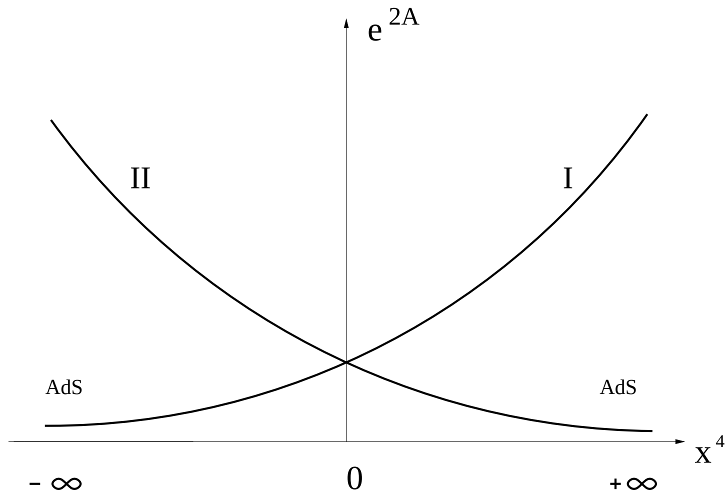

Figure 4: Solutions for branes ([18]): Warp factor as a function of x^4; for solution I, the warp factor behaves as an exponential (AdS) at $-\infty$ and as a polynomial around $+\infty$. Solution II can be obtained from solution I by $x^4 \to -x^4$.

As an example, in fig. 4, we plot the first branch solutions (I and II) for the warp factor found in [18]. They correspond to $p_1 < 0$ and $p_1 > 0$ respectively, i.e. solution II can be obtained from solution I by a reflection around the origin $x^4 = 0$. Solution I corresponds to patching together in a smooth way the above found solutions for

$\varphi \to \varphi_0$ and $\varphi \to -\infty$ in which case the singularity encountered in the $\varphi \to +\infty$ regime is avoided. Solutions I and II individually do not possess reflection symmetry around the origin, but they are the ones appropriate for constructing models with explicit branes inserted in the bulk. More specifically, we can construct a brane world scenario as follows: Consider the solutions of fig. 4 and choose a region in the x^4 coordinate, symmetric around x^4. The region is, say, the interval $[-\pi R, +\pi R]$. If $-\pi R < x^4 < 0$, take solution I as it is and if $0 \leq x^4 \leq +\pi R$, take solution II to get a Z_2 symmetric situation. The orbifold will then be the region $0 < x^4 < \pi R$. A one brane world can be obtained by taking R to infinity. This construction, for the warp factor, is illustrated in fig. 5.

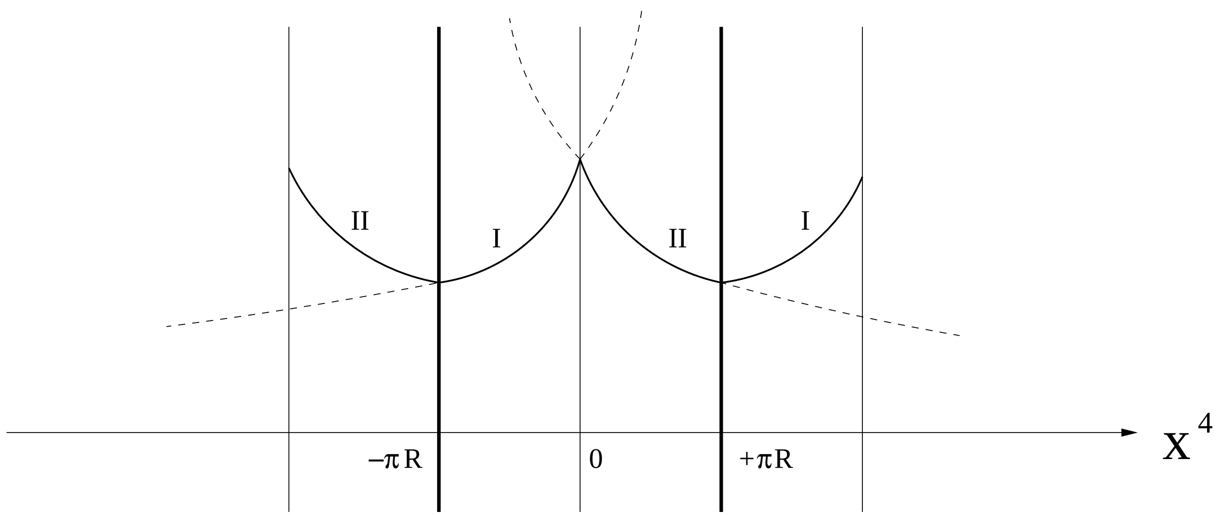

Figure 5: The orbifold construction.

With the $D3$-branes present, the 5 dimensional action is modified to:

$$S(\varphi) = \int d^5x\sqrt{-G_5}\Big[\mathcal{R} - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\Big] - \int d^4x\sqrt{-G_4^{(1)}}T^{(1)}(\varphi) -$$

$$- \int d^4x\sqrt{-G_4^{(2)}}T^{(2)}(\varphi), \tag{3.23}$$

where $G^{(i)}_{4\mu\nu}$ is the induced metric on the (i)'th brane by $G_{5\mu\nu}$, and for our metric it is simply $G^{(i)}_{4\mu\nu} = \eta_{\mu\nu}$, and $T^{(1)}(\varphi)$ and $T^{(2)}(\varphi)$ are the tensions of the branes. The dependence of T on φ is kept arbitrary since it will in general be affected by quantum effects on the brane when supersymmetry is broken. The action for the branes is written in the static gauge so that the embedding functions are $x^\mu(\xi) = \xi^\mu$, $\mu = 0, \cdots, 3$ and we ignore their fluctuations. Also of course we have set all other fields on the brane to the minimum of the quantum effective action so that what we

have kept is just an effective description of the ground state of the theory on the brane.

Now, in addition to the equations of motion, we have to satisfy the jump conditions at $x^4 = 0$ (and $x^4 = \pi R$ in the two brane case), which constitute the connection between the two solutions at opposite sides of the brane(s). They are:

$$2A'(x^4) = +\frac{1}{6}T^{(1)}(\varphi(x^4)) \mid_{x^4=0}, \tag{3.24}$$

$$2A'(x^4) = -\frac{1}{6}T^{(2)}(\varphi(x^4)) \mid_{x^4=\pi R}, \tag{3.25}$$

$$2\varphi'(x^4) = -\frac{\partial T^{(1)}}{\partial \varphi}(\varphi(x^4)) \mid_{x^4=0}, \tag{3.26}$$

$$2\varphi'(x^4) = +\frac{\partial T^{(2)}}{\partial \varphi}(\varphi(x^4)) \mid_{x^4=\pi R} . \tag{3.27}$$

3.1 More General Solutions

In a two-brane world, we need both nontrivial integration constants of the second order equations of motion (3.2), (3.3) and (3.4), because we have only R_5 and the size of the orbifold R to satisfy four jump conditions. One might think a solution with two branes and a pure AdS bulk is possible [10], since there are only two non trivial jump conditions to satisfy (one for each brane) and two available constants, R_5 and R. However, as was pointed out in [3], the two branes have equal and opposite tensions at every point of the RG flow and the tension for a flat brane solution is fixed only in terms of R_5. The orbifold size does not enter the jump conditions and therefore it can not be used. In order to have a solution without fine tuning, we need a nontrivial scalar field in the bulk. But then, our initial choice of W does not provide us with the two required integration constants, since as we explained, we have lost one integration constant in the choice (3.8). We therefore look for more general solutions to the equations of motion.

We make the ansatz

$$\varphi = \varphi_0 + \epsilon f_1(x^4) + \epsilon^2 f_2(x^4) + \epsilon^3 f_3(x^4) + \cdots \tag{3.28}$$

$$A = kx^4 + \epsilon g_1(x^4) + \epsilon^2 g_2(x^4) + \epsilon^3 g_3(x^4) + \cdots \tag{3.29}$$

with $k > 0$, and we substitute it into (3.2), (3.3) and (3.4). We obtain an infinite set of differential equations. We show the result for (3.2) and (3.3), up to order ϵ^3:

$$f_1'' + 4kf_1' + pf_1 = 0, \quad g_1'' = 0 \tag{3.30}$$

$$f_2'' + 4kf_2' + pf_2 - \frac{1}{2}a(a^2 - b^2)R_5 e^{\frac{1}{2}a\varphi_0} f_1^2 + 4f_1'g_1' = 0, \quad g_2'' + \frac{1}{6}f_1'^2 = 0 \tag{3.31}$$

$$f_3'' + 4kf_3' + pf_3 - a(a^2 - b^2)R_5 e^{\frac{1}{2}a\varphi_0} f_1 f_2 - \frac{1}{6}a(a^3 - b^3)R_5 e^{\frac{1}{2}a\varphi_0} f_1^3 +$$

$$+4f_1' g_2' + 4f_2' g_1' = 0, \quad g_3'' + \frac{1}{3}f_1' f_2' = 0, \tag{3.32}$$

where $p = -32k^2$. The above pattern (that presumably continues), suggests that we can write the general solution for the field φ as:

$$\varphi(x^4) = \varphi_0 + c_1^{\varphi} e^{4kx^4} + c_2^{\varphi} e^{-8kx^4} + P(x^4, c_1^{\varphi}, c_2^{\varphi}), \tag{3.33}$$

where the part of the expression with the exponentials is the general solution to the equation $f''+4kf'+pf = 0$ and $P(x^4, c_1^{\varphi}, c_2^{\varphi})$ is some function of x^4 containing also the two integration constants and ϵ has been absorbed into the integration constants. By looking at (3.30), (3.31) and (3.32), one can see that the solution is a small deviation from AdS in two cases. One case is when $x^4 \to -\infty$. Then, the expansion in ϵ makes sense only if $c_2^{\varphi} = 0$ and therefore this case is just the solution (3.15). The second case is when $x^4 \simeq 0$ and $c_1^{\varphi}, c_2^{\varphi} << 1$. This is a satisfactory solution with two explicit integration constants as long as the jump conditions allow them to be small. Another observation we can make here is that when $c_2^{\varphi} \neq 0$, it is not clear that φ has two separate branches. Nevertheless, we can still apply the method for the construction of the orbifold with the only caveat that if the jump conditions require integration constants of order one or larger, we still do not have an explicit solution.

We will now find a solution to the second order equations of motion in the $\varphi < \varphi_0$ region which will be valid for a larger range of the integration constants. Define $H \equiv A'$ and denote $\frac{\partial}{\partial\varphi}$ with a dot. Also, for simplicity we will assume that $R_5 = 8m^2 = 1$, even though we have to keep in mind that R_5 is really a parameter determined by the integration constants. Combining equations (3.3) and (3.4), we obtain the equation $\dot{H}^2 = \frac{1}{36}(24H^2 + 2V)$. Let us assume that $24H^2 >> 2|V|$. We will verify soon that this is a valid assumption in the regime of interest. Then, we can easily solve for H: $H = e^{\sqrt{\frac{2}{3}}\varphi}$, where we have chosen the positive branch of the square root. Using the value of φ_0 from the minimization of V, we deduce that $2V$ could be consistently dropped provided that

$$\frac{1}{12} e^{a\varphi}\Big(1 - \frac{a}{b} e^{(b-a)(\varphi-\varphi_0)}\Big) << e^{2\sqrt{\frac{2}{3}}\varphi}. \tag{3.34}$$

Since our solution corresponds to $\varphi \in (-\infty, \varphi_0)$, a numerical estimate tells us that the above is true if, approximately, $-3 < \varphi < -0.6$ (for $R_5 = 8m^2 = 1$, we get from (3.13) $\varphi_0 \simeq -0.6$). Next, using the above solution for H, equation (3.2) becomes $\varphi'' + 4e^{\sqrt{\frac{2}{3}}\varphi}\varphi' - \dot{V} = 0$. In the $\varphi < \varphi_0$ region the potential V approaches zero exponentially and its shape in this regime is rather flat, which means that $\dot{V} \simeq 0$. Indeed, one can neglect $\dot{V}$ in the equation for φ provided that

$$| - ae^{a\varphi} + be^{b\varphi}| << \; 4e^{\sqrt{\frac{2}{3}}\varphi}|\varphi'| \quad \text{and} \quad |\varphi''|. \tag{3.35}$$

Then we can solve for φ and we obtain

$$\varphi(x^4) = \frac{\sqrt{6}}{2} \ln \frac{c_2^\varphi}{1 + 12 e^{\frac{\sqrt{6}}{3} c_2^\varphi (c_1^\varphi + x^4)}} + c_2^\varphi (c_1^\varphi + x^4) + \frac{\sqrt{6}}{4} \ln 6. \tag{3.36}$$

By computing φ' and φ'' from the above expression, one can verify that (3.35) is true in the relevant range of φ for a large range of c_1^φ and c_2^φ, when $x^4 \sim \mathcal{O}(0-10)$. [6]

For $\varphi >> \varphi_0$, we can take similar steps. We can again assume for simplicity that $R_5 = 8m^2 = 1$. The derivative of the warp factor is obtained by solving the equation $\dot{H} = \frac{\sqrt{2}}{6} e^{\frac{b}{2}\varphi}$, which yields $H = \frac{\sqrt{2}}{3b} e^{\frac{b}{2}\varphi}$ up to an irrelevant integration constant. The equation for φ becomes $\varphi'' = b e^{b\varphi}$, where we have ignored the smaller terms. The solution for φ is then

$$\varphi(x^4) = \frac{\sqrt{15}}{10} \ln \Big[-\frac{c_1^\varphi}{2} [1 + \tan \Big(-c_1^\varphi \frac{5}{3} (x^4 + c_2^\varphi)^2 \Big)] \Big]. \tag{3.37}$$

Finally, we assure that one can solve the equations of motion in an analogous fashion for arbitrary R_5.

We can now construct the orbifold as in fig.5 and satisfy the four jump conditions. An interesting fact is that to do so, along with c_1^φ and c_2^φ, we have to use R_5 and R, which provides us with a RG scale dependent determination of the $5D$ dilaton and the orbifold size.

4. Conclusions

In this paper we have suggested a string theoretic framework for a two brane scenario. The main issue we have addressed is the possibility of obtaining a flat four dimensional world on the branes. The supersymmetric case is a degenerate one in which the matching conditions (equations (3.24-3.27)) are automatically satisfied. When supersymmetry is broken however these conditions are non-trivial. The solutions to the equations of motion and constraint have two independent integration constants. In addition there is the distance between the branes yielding in general three adjustable constants in all. As was observed in [8] this would mean that there would have to be a tunable parameter either in the bulk potential or the brane tension.

The main observation of this paper is that there is a possible string construction in which the appearence of such a tunable parameter in the five dimensional theory is manifest. Indeed at the ten dimensional level there is no tuning, the parameter appears in the compactification and is in this sense an integration constant. Its appearence is related to the fact that we are dealing with a Ricci non-flat compactification (which gives a potential with a critical point for the breathing mode).

[6] (3.35) is in fact true for any finite x^4 for appropriate choice of the order of magnitude of the integration constants, but the regime around $x^4 = 0$ is the interesting one since it is where the branes are located.

Solutions in the presence of supersymmetry breaking necessarily must have the complete set of free parameters that we have enumerated above. In particular the solutions for the warp factor and the breathing mode must contain the two independent integration constants. Unfortunately it was not possible to obtain exact analytic solutions displaying this but we have shown how to obtain approximate solutions that contain the two integration constants. In other words we have demonstrated the existence of (approximate) solutions that can be used to support flat branes after supersymmetry breaking, justifying the picture illustrated in figure 5.

There is one major unsolved problem though. What we have done strictly speaking is to demonstrate the existence of a flat two brane scenario after supersymmetry breaking in the context of type IIB supergravity. In the paper we have suggested that this ought to imply that there is a corresponding microscopic i.e. string theoretic implementation but this was not explicitly demonstrated. In particular in the supersymmetric situation, the D-brane tension in five dimensions should be obtained from the usual formula for D-brane tension in ten dimensions. However, as was demonstrated in recent papers [7] [19][20], five dimensional supersymmetry requires that the tension $T(\varphi)$ is essentially the superpotential W of equation (3.8). It is not at all clear how this comes from the usual ten dimensional action for the D3 brane though it is possible that this action needs to be modified for string theory in the presence of RR flux and Ricci-non-flat compactifications. Perhaps one should expect a simple relation to the 10 dimensional tension only at the maximally supersymmetric point, the $\mathcal{N}$=8 critical point (2.3). At this point one finds that the tension in five dimensions $T(\varphi_0) = \frac{3}{4}T_{D_3}$, a relation that was first found by [21]. However this apparently can be justified from the string point of view [20]. Perhaps this is all that is needed, but the matter is not entirely clear to us and is currently under investigation.

The phenomenological importance of our scenario is that we can have a $\mathcal{N} = 2$ bulk five dimensional theory (by compactifying on a squashed five sphere as for example in [22], so that the brane theory would be $\mathcal{N} = 1$ in four dimensions. The implication in four dimensions is that we have an adjustable constant (essentially an integration constant) in the superpotential that can be used to set the cosmological constant to zero after supersymmetry breaking (say by gaugino condensation). If this scenario can be completely justified (i.e. if the problem mentioned above can be solved) then it would be the first time that a string theoretic justification could be given for adding an adjustable constant to the superpotential.

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[7]These appeared after the first version of this paper was published on the e-print archive hep-th.

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